1 Resolver por la regla de Cramer:

$$\begin{cases} 2x - y + z = 3 \\ 2y - z = 1 \\ -x + y = 1 \end{cases} \qquad \begin{cases} x + y + z = 1 \\ x - y + z = 1 \\ -x + y + z = 1 \end{cases}$$

2 Discutir y resolver, si es posible, el sistema:

$$\begin{cases} x + 2y - 2z = 10 \\ 4x - y + z = 4 \\ -2x + y + z = -2 \\ -x - 3y = -11 \end{cases} \qquad \begin{cases} x + y - z + u + v = 2 \\ x - 2y + u = 5 \\ -x + z + \quad + 2v = 3 \\ 3y + z - 2u = -1 \end{cases}$$

3 Resolver el sistema homogéneo:

$$\begin{cases} 3x - 2y - z = 0 \\ -4x + y - z = 0 \\ 2x + 2z = 0 \end{cases}$$

4 Discutir y resolver el sistema cuando sea compatible.

$$\begin{cases} 2x + y + az = 4 \\ x + z = 2 \\ x + y + z = 2 \end{cases} \qquad \begin{cases} x + y + z = a \\ x + (a+1)y + z = 2a \\ x + y + (1+a)z = 0 \end{cases}$$

$$\begin{cases} ax + z + t = 1 \\ ay + z - t = 1 \\ ay + z - 2t = 2 \\ az - t = 0 \end{cases}$$

$$\begin{cases} (a+1)x + y + z = a+1 \\ x + (a+1)y + z = a+3 \\ x + y + (1+a)z = -2a-4 \end{cases}$$

$$\begin{cases} ax + y - z = 0 \\ x + 3y + z = 0 \\ 3x + 10y + 4z = 0 \end{cases} \qquad \begin{cases} 2x - y + z - 2t = -5 \\ 2x + 2y - 3z + t = -1 \\ -x + y - z = -1 \\ 4x - 3y + 2z - 3t = -8 \end{cases}$$

5 Estudiar los siguientes sistemas según los distintos valores de a y b.

$$\begin{cases} 2x - y + z = 3 \\ x - y + z = 2 \\ 3x - y - az = b \end{cases}$$

6 Determinar para qué valores de k, el siguiente sistema tiene infinitas soluciones.

$$\left. \begin{aligned} x + y + z &= 0 \\ x - y + z &= 0 \\ kx + z &= 0 \end{aligned} \right\}$$

SOLUCIONES

Ejercicio 1.-

1

$$\Delta = \begin{vmatrix} 2 & -1 & 1 \\ 0 & 2 & -1 \\ -1 & 1 & 0 \end{vmatrix} = 3 \qquad x = \frac{\begin{vmatrix} 3 & -1 & 1 \\ 1 & 2 & -1 \\ 1 & 1 & 0 \end{vmatrix}}{3} = \frac{3}{3} = 1$$

$$y = \frac{\begin{vmatrix} 2 & 3 & 1 \\ 0 & 1 & -1 \\ -1 & 1 & 0 \end{vmatrix}}{3} = \frac{6}{3} = 2 \qquad z = \frac{\begin{vmatrix} 2 & -1 & 3 \\ 0 & 2 & 1 \\ -1 & 1 & 1 \end{vmatrix}}{3} = \frac{9}{3} = 3$$

2

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \end{vmatrix} = -4 \qquad x = \frac{\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & 1 \end{vmatrix}}{-4} = \frac{0}{-4} = 0 \qquad y = \frac{\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ -1 & 1 & 1 \end{vmatrix}}{-4} = \frac{0}{-4} = 0$$

$$z = \frac{\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \end{vmatrix}}{-4} = \frac{-4}{-4} = 1$$

Ejercicio 2.-

1

$$A = \begin{pmatrix} 1 & 2 & -2 \\ 4 & -1 & 1 \\ -2 & 1 & 1 \\ -1 & -3 & 0 \end{pmatrix} \qquad A' = \begin{pmatrix} 1 & 2 & -2 & 10 \\ 4 & -1 & 1 & 4 \\ -2 & 1 & 1 & -2 \\ -1 & -3 & 0 & -11 \end{pmatrix}$$

$$|1| = 1 \neq 0 \qquad \begin{vmatrix} 1 & 2 \\ 4 & -1 \end{vmatrix} = -7 \neq 0 \qquad \begin{vmatrix} 1 & 2 & -2 \\ 4 & -1 & 1 \\ -2 & 1 & 1 \end{vmatrix} = -18 \neq 0$$

$$\begin{vmatrix} 1 & 2 & -2 & 10 \\ 4 & -1 & 1 & 4 \\ -2 & 1 & 1 & -2 \\ -1 & -3 & 0 & -11 \end{vmatrix} = 0 \qquad r(A) = 3 \qquad r(A') = 3 \qquad n = 3$$

Sistema compatible determinado

$$x = \frac{\begin{vmatrix} 10 & 2 & -2 \\ 4 & -1 & 1 \\ -2 & 1 & 1 \end{vmatrix}}{-18} = \frac{-36}{-18} = 2 \qquad y = \frac{\begin{vmatrix} 1 & 10 & -2 \\ 4 & 4 & 1 \\ -2 & -2 & 1 \end{vmatrix}}{-18} = \frac{-54}{-18} = 3$$

$$z = \frac{\begin{vmatrix} 1 & 2 & 10 \\ 4 & -1 & 4 \\ -2 & 1 & -2 \end{vmatrix}}{-18} = \frac{18}{-18} = -1$$

2

$$A = \begin{pmatrix} 1 & 1 & -1 & 1 & 1 \\ 1 & -2 & 0 & 1 & 0 \\ -1 & 0 & 1 & 0 & 2 \\ 0 & 3 & 1 & -2 & 0 \end{pmatrix} \qquad A' = \begin{pmatrix} 1 & 1 & -1 & 1 & 1 & 2 \\ 1 & -2 & 0 & 1 & 0 & 5 \\ -1 & 0 & 1 & 0 & 2 & 3 \\ 0 & 3 & 1 & -2 & 0 & -1 \end{pmatrix}$$

$$|1| \neq 0 \qquad \begin{vmatrix} 1 & 1 \\ 1 & -2 \end{vmatrix} \neq 0 \qquad \begin{vmatrix} 1 & 1 & -1 \\ 1 & -2 & 0 \\ -1 & 0 & 1 \end{vmatrix} \neq 0$$

$$\begin{vmatrix} 1 & 1 & -1 & 1 \\ 1 & -2 & 0 & 1 \\ -1 & 0 & 1 & 0 \\ 0 & 3 & 1 & -2 \end{vmatrix} = 8 \neq 0$$

$r(A) = 4$ $\qquad r(A') = 4$ $\qquad n = 5$

Sistema compatible indeterminado

$$\begin{cases} x + y - z + u & = 2 - \lambda \\ \quad x - 2y + u & = 5 \\ -x \quad + z + & = 3 - 2\mu \\ \quad 3y + z - 2u & = -1 \end{cases} \qquad v = \lambda \qquad x = \frac{\begin{vmatrix} 2-\lambda & 1 & -1 & 1 \\ 5 & -2 & 0 & 1 \\ 3-2\lambda & 0 & 1 & 0 \\ -1 & 3 & 1 & -2 \end{vmatrix}}{8} = \frac{18 + 3\lambda}{8}$$

$$y = \frac{\begin{vmatrix} 1 & 2-\lambda & -1 & 1 \\ 1 & 5 & 0 & 1 \\ -1 & 3-2\lambda & 1 & 0 \\ 0 & -1 & 1 & -2 \end{vmatrix}}{8} = \frac{6 - 7\lambda}{8} \qquad z = \frac{\begin{vmatrix} 1 & 1 & 2-\lambda & 1 \\ 1 & -2 & 5 & 1 \\ -1 & 0 & 3-2\lambda & 0 \\ 0 & 3 & -1 & -2 \end{vmatrix}}{8} = \frac{42 - 13\lambda}{8}$$

$$u = \frac{\begin{vmatrix} 1 & 1 & -1 & 2-\lambda \\ 1 & -2 & 0 & 5 \\ -1 & 0 & 1 & 3-2\lambda \\ 0 & 3 & 1 & -1 \end{vmatrix}}{8} = \frac{34 - 17\lambda}{8}$$

Ejercicio 3.-

$$\begin{vmatrix} 3 & -2 & -1 \\ -4 & 1 & -1 \\ 2 & 0 & 2 \end{vmatrix} \neq 0$$

r = 3 n = 3 Solución trivial: $x = y = z = 0$

Ejercicio 4.-

1

$$A = \begin{pmatrix} 2 & 1 & a \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix} \qquad A' = \begin{pmatrix} 2 & 1 & a & 4 \\ 1 & 0 & 1 & 2 \\ 1 & 1 & 1 & 2 \end{pmatrix} |A| - \begin{vmatrix} 2 & 1 & d \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{vmatrix} - a - 2 \qquad a - 2 - 0 \qquad a - 2$$

$$A' = \begin{pmatrix} 2 & 1 & a & 4 \\ 1 & 0 & 1 & 2 \\ 1 & 1 & 1 & 2 \end{pmatrix} c_4 \to 2c_1 \qquad A' = \begin{pmatrix} 2 & 1 & a \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

Si $a = 2$ $\qquad r(A) = r(A') = 2$ $\qquad n = 3$

$$\begin{cases} 2x + y = 4 - 2\lambda \\ x \quad = 2 - \lambda \end{cases} \qquad z = \lambda \qquad \Delta = \begin{vmatrix} 2 & 1 \\ 1 & 0 \end{vmatrix} = -1$$

Si $a \neq 2$ $\qquad r(A) = r(A') = n = 3$

Sistema compatible determinado

$$\Delta_1 = \begin{vmatrix} 4 & 1 & a \\ 2 & 0 & 1 \\ 2 & 1 & 1 \end{vmatrix} = 2a - 4; \qquad \Delta_2 = \begin{vmatrix} 2 & 4 & a \\ 1 & 2 & 1 \\ 1 & 2 & 1 \end{vmatrix} = 0; \quad \Delta_3 = \begin{vmatrix} 2 & 1 & 4 \\ 1 & 0 & 2 \\ 1 & 1 & 2 \end{vmatrix} = 0$$

$$x = \frac{2a - 4}{a - 2} = 2 \qquad y = 0 \qquad z = 0$$

2

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & a+1 & 1 \\ 1 & 1 & a+1 \end{pmatrix} \qquad A' = \begin{pmatrix} 1 & 1 & 1 & a \\ 1 & a+1 & 1 & 2a \\ 1 & 1 & a+1 & 0 \end{pmatrix} \quad |A| = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a+1 & 1 \\ 1 & 1 & a+1 \end{vmatrix} = a^2$$

Si $a = 0$

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \qquad A' = \begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

$r(A) = r(A') = 1$ $\qquad n = 3$

Sistema compatible indeterminado

$x+y+z=0$ $\qquad y = \lambda$ $\qquad z = \mu$ $\qquad x = -\lambda - \mu$

Si $a \neq 0,$ $r(A) = r(A') = n = 3$

Sistema compatible determinado

$$\Delta_1 = \begin{vmatrix} a & 1 & 1 \\ 2a & a+1 & 1 \\ 0 & 1 & a+1 \end{vmatrix} = a^3; \quad \Delta_2 = \begin{vmatrix} 1 & a & 1 \\ 1 & 2a & 1 \\ 1 & 0 & a+1 \end{vmatrix} = a^2; \quad \Delta_3 = \begin{vmatrix} 1 & 1 & a \\ 1 & a+1 & 2a \\ 1 & 1 & 0 \end{vmatrix} = -a^2$$

$$x = \frac{a^3}{a^2} = a \qquad y = \frac{a^2}{a^2} = 1 \qquad z = \frac{-a^2}{a^2} = -1$$

3

$$A = \begin{pmatrix} a+1 & 1 & 1 \\ 1 & a+1 & 1 \\ 1 & 1 & a+1 \end{pmatrix} \qquad A' = \begin{pmatrix} a+1 & 1 & 1 & 1 \\ 1 & a+1 & 1 & b \\ 1 & 1 & a+1 & b^2 \end{pmatrix}$$

$$|A| = \begin{vmatrix} a+1 & 1 & 1 \\ 1 & a+1 & 1 \\ 1 & 1 & a+1 \end{vmatrix} = a^2(a+3) \qquad a=0 \qquad a=-3$$

Si $\begin{cases} a \neq 0 \\ \forall b \end{cases}$ $r(A) = r(A') = n = 3$

Sistema compatible determinado

Si $a = 0$

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \qquad A' = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & b \\ 1 & 1 & 1 & b^2 \end{pmatrix} \quad |A'| = \begin{vmatrix} 1 & 1 \\ 1 & b \end{vmatrix} = b-1 \qquad b=1$$

Si $\begin{cases} a = 0 \\ b \neq 1 \end{cases}$ $r(A) = 1 \quad r(A') = 2$

Sistema incompatible

Si $\begin{cases} a = 0 \\ b = 1 \end{cases}$ $r(A) = 1 \quad r(A') = 1$

Sistema compatible indeterminado

Si $a = -3$

$$A = \begin{pmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{pmatrix} \quad A' = \begin{pmatrix} -2 & 1 & 1 & 1 \\ 1 & -2 & 1 & b \\ 1 & 1 & -2 & b^2 \end{pmatrix} |A| = \begin{vmatrix} -2 & 1 \\ 1 & -2 \end{vmatrix} \neq 0 \qquad |A'| = \begin{vmatrix} -2 & 1 & 1 \\ 1 & -2 & b \\ 1 & 1 & b^2 \end{vmatrix} = 3(b^2+b+1)$$

Si $\begin{cases} a = 0 \\ b = 1 \end{cases}$ $r(A) = 1 \quad r(A') = 1$

Sistema incompatible

4

$$A = \begin{pmatrix} a & 0 & 1 & 1 \\ 0 & a & 1 & -1 \\ 0 & a & 1 & -2 \\ 0 & 0 & a & -1 \end{pmatrix} \quad A' = \begin{pmatrix} a & 0 & 1 & 1 & 1 \\ 0 & a & 1 & -1 & 1 \\ 0 & a & 1 & -2 & 2 \\ 0 & 0 & a & -1 & 0 \end{pmatrix} |A| = \begin{vmatrix} a & 0 & 1 & 1 \\ 0 & a & 1 & -1 \\ 0 & a & 1 & -2 \\ 0 & 0 & a & -1 \end{vmatrix} = a^3 \qquad a=0$$

Si $a = 0$

$$A = \begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad A' = \begin{pmatrix} 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & -2 & 2 \\ 0 & 0 & 0 & -1 & 0 \end{pmatrix} \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} \neq 0 \qquad \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & -2 & 2 \end{vmatrix} \neq 0$$

$r(A) = 2 \qquad r(A') = 3$ Sistema incompatible

Si $a \neq 0$ $r(A) = r(A') = n = 4$

Sistema compatible determinado

$$\begin{cases} ax & & +z & +t & = & 1 \\ & ay & +z & -t & = & 1 \\ & ay & +z & -2t & = & 2 \\ & & +az & -t & = & 0 \end{cases} \quad f_3 - f_2, \quad \begin{cases} ax & & +z & +t & = & 1 \\ & ay & +z & -t & = & 1 \\ & & & -t & = & 1 \\ & & +az & -t & = & 0 \end{cases}$$

$x=\frac{2a+1}{a^2}$ $y=\frac{1}{a^2}$ $z=\frac{-1}{a}$ $t=-1$

5

$$|A|=\begin{vmatrix} a & 1 & -1 \\ 1 & 3 & 1 \\ 3 & 10 & 4 \end{vmatrix}=2(a-1) \quad a=1$$

$a \neq 1$ r = 3 n = 3

Solución trivial: $x=y=z=0$

Si $a=1$ $r=2$ $n=3$

Sistema compatible indeterminado

$$\begin{cases} x+y=\lambda \\ x+3y=-\lambda \end{cases} \quad z=\lambda$$

$$\Delta=\begin{vmatrix} 1 & 1 \\ 1 & 3 \end{vmatrix}=2 \quad \Delta_1=\begin{vmatrix} \lambda & 1 \\ -\lambda & 3 \end{vmatrix}=4\lambda \quad \Delta_2=\begin{vmatrix} 1 & \lambda \\ 1 & -\lambda \end{vmatrix}=-2\lambda$$

$$x=\frac{4\lambda}{2}=2\lambda \quad y=\frac{-2\lambda}{2}=-\lambda \quad z=\lambda$$

6

$$A=\begin{pmatrix} 2 & -1 & 1 & -2 \\ 2 & 2 & -3 & 1 \\ -1 & 1 & -1 & 0 \\ 4 & -3 & 2 & -3 \end{pmatrix} \quad A'=\begin{pmatrix} 2 & -1 & 1 & -2 & -5 \\ 2 & 2 & -3 & 1 & -1 \\ -1 & 1 & -1 & 0 & -1 \\ 4 & -3 & 2 & -3 & -8 \end{pmatrix}$$

$$|2|\neq 0 \quad \begin{vmatrix} 2 & -1 \\ 2 & 2 \end{vmatrix}\neq 0 \quad \begin{vmatrix} 2 & -1 & 1 \\ 2 & 2 & -3 \\ -1 & 1 & -1 \end{vmatrix}\neq 0 \quad \begin{vmatrix} 2 & -1 & 1 & -2 \\ 2 & 2 & -3 & 1 \\ -1 & 1 & -1 & 0 \\ 4 & -3 & 2 & -3 \end{vmatrix}=-10$$

$r(A)=4$ $r(A')=4$ $n=4$

Sistema compatible determinado

Ejercicio 5.-

$$A=\begin{pmatrix} 2 & -1 & 1 \\ 1 & -1 & 1 \\ 3 & -1 & -a \end{pmatrix} \quad A'=\begin{pmatrix} 2 & -1 & 1 & 3 \\ 1 & -1 & 1 & 2 \\ 3 & -1 & -a & b \end{pmatrix} |A|=\begin{vmatrix} 2 & -1 & 1 \\ 1 & -1 & 1 \\ 3 & -1 & -a \end{vmatrix}=1+a \quad 1+a=0 \quad a=-1$$

Si $a \neq -1$ $r(A)=r(A')=n=3$ Sistema compatible determinado

Si $a=-1$

$$|A|=\begin{vmatrix} 2 & -1 \\ 1 & -1 \end{vmatrix}\neq 0 \quad |A'|=\begin{vmatrix} 2 & -1 & 3 \\ 1 & -1 & 2 \\ 3 & -1 & b \end{vmatrix}=4-b=0 \quad b=4$$

Si $\begin{cases} a=-1 \\ b\neq 4 \end{cases}$ $r(A)=2$ $r(A')=3$ Sistema incompatible

Si $\begin{cases} a=-1 \\ b=4 \end{cases}$ $r(A)=2$ $r(A')=2$ Sistema compatible indeterminado

Ejercicio 6.-

$$\begin{pmatrix} 1 & 1 & 1 & \vdots & 0 \\ 1 & -1 & 1 & \vdots & 0 \\ k & 0 & 1 & \vdots & 0 \end{pmatrix} \xrightarrow{f_2+f_1} \begin{pmatrix} 1 & 1 & 1 & \vdots & 0 \\ 2 & 0 & 2 & \vdots & 0 \\ k & 0 & 1 & \vdots & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & 1 & \vdots & 0 \\ 1 & 0 & 1 & \vdots & 0 \\ k & 0 & 1 & \vdots & 0 \end{pmatrix} \xrightarrow{f_3-f_1} \begin{pmatrix} 1 & 1 & 1 & \vdots & 0 \\ 1 & 0 & 1 & \vdots & 0 \\ k-1 & 0 & 0 & \vdots & 0 \end{pmatrix}$$

$k-1=0$ $k=1$ Sistema Compatible Indeterminado

$$\left.\begin{matrix} x+y+z=0 \\ x+z=0 \end{matrix}\right\} \quad \begin{matrix} x=\lambda \\ y=0 \\ z=-\lambda \end{matrix}$$