

MATEMATICAS. 4ºESO-B. TEMA 5: Sistemas de Ecuaciones

1.- Resuelve los siguientes sistemas de ecuaciones por el **método de sustitución**:

$$\begin{array}{lll}
 1. \begin{cases} 3x - 4y = -6 \\ 2x + 4y = 16 \end{cases} & 2. \begin{cases} 2x + 3y = -1 \\ 3x + 4y = 0 \end{cases} & 3. \begin{cases} \frac{x+3y}{2} = 5 \\ 3x - y = 5y \end{cases} \\
 4. \begin{cases} \frac{x+3y}{2} = 5 \\ 4 - \frac{2x-y}{2} = 1 \end{cases} & 5. \begin{cases} x + y = 60 \\ 16x + 20y = 1100 \end{cases} & 6. \begin{cases} 3x + 2y = 7 \\ 4x - 3y = -2 \end{cases}
 \end{array}$$

2.- Resuelve los siguientes sistemas de ecuaciones por el **método de igualación**:

$$\begin{array}{lll}
 1. \begin{cases} 3x - 4y = -6 \\ 2x + 4y = 16 \end{cases} & 2. \begin{cases} 2x + 3y = -1 \\ 3x + 4y = 0 \end{cases} & 3. \begin{cases} 3x + 2y = 7 \\ 4x - 3y = -2 \end{cases} \\
 4. \begin{cases} \frac{x+3y}{2} = 5 \\ 3x - y = 5y \end{cases} & 5. \begin{cases} \frac{x+3y}{2} = 5 \\ 4 - \frac{2x-y}{2} = 1 \end{cases} & 6. \begin{cases} x + y = 60 \\ 16x + 20y = 1100 \end{cases}
 \end{array}$$

3.- Resuelve los siguientes sistemas de ecuaciones por el **método de reducción**:

$$\begin{array}{llll}
 1. \begin{cases} 3x - 4y = -6 \\ 2x + 4y = 16 \end{cases} & 2. \begin{cases} 3x + 2y = 7 \\ 4x - 3y = -2 \end{cases} & 3. \begin{cases} 3x + 2y = 24 \\ x + 3y = 3 \end{cases} & 4. \begin{cases} x + y = 3500 \\ x - \frac{10x}{100} + y - \frac{8y}{100} = 3170 \end{cases} \\
 5. \begin{cases} 2x + 3y = -1 \\ 3x + 4y = 0 \end{cases} & 6. \begin{cases} \frac{x+y}{2} = x - 1 \\ \frac{x-y}{2} = y + 1 \end{cases} & 7. \begin{cases} x + y = 2000 \\ x + \frac{10x}{100} + y + \frac{15y}{100} = 2260 \end{cases} & 8. \begin{cases} x + y = 58 \\ 2x + 4y = 168 \end{cases}
 \end{array}$$

4.- Resuelve los siguientes **sistemas de ecuaciones**:

1. Resuelve gráficamente los sistemas:

$$\begin{array}{ll}
 a) \begin{cases} 2x + 3y = -1 \\ 3x + 4y = 0 \end{cases} & b) \begin{cases} 3x + 2y = 7 \\ 4x - 3y = -2 \end{cases}
 \end{array}$$

2. Halla por el método que quieras la solución de los siguientes sistemas:

$$\begin{array}{lll}
 a) \begin{cases} \frac{x+y}{2} = x - 1 \\ \frac{x-y}{2} = y + 1 \end{cases} & b) \begin{cases} \frac{x+3y}{2} = 5 \\ 3x - y = 5y \end{cases} & c) \begin{cases} \frac{x+3y}{2} = 5 \\ 4 - \frac{2x-y}{2} = 1 \end{cases} \\
 d) \begin{cases} \frac{x}{2} + \frac{y}{3} = 4 \\ \frac{x}{3} + y = 1 \end{cases} & e) \begin{cases} \frac{x+1}{3} + \frac{y-1}{2} = 0 \\ \frac{x+2y}{3} - \frac{x+y+2}{4} = 0 \end{cases}
 \end{array}$$

5.- Resuelve los siguientes **sistemas de ecuaciones no lineales**:

$$\begin{array}{llll}
 1. \begin{cases} x^2 + y^2 = 25 \\ x + y = 7 \end{cases} & 2. \begin{cases} x + y = 7 \\ x \cdot y = 12 \end{cases} & 3. \begin{cases} x^2 + y^2 = 169 \\ x + y = 17 \end{cases} & 4. \begin{cases} y^2 - 2y + 1 = x \\ \sqrt{x} + y = 5 \end{cases} & 5. \begin{cases} \frac{1}{x^2} + \frac{1}{y^2} = 13 \\ \frac{1}{x} - \frac{1}{y} = 1 \end{cases}
 \end{array}$$

SOLUCIONES

Ejercicio n° 1.

$$1. \begin{cases} 3x - 4y = -6 \\ 2x + 4y = 16 \end{cases} \quad 2x = 16 - 4y \quad x = 8 - 2y$$

$$3(8 - 2y) - 4y = -6 \quad 24 - 6y - 4y = -6 \quad -10y = -30 \quad y = 3$$

$$x = 8 - 2 \cdot 3 = 8 - 6 \quad x = 2 \quad x = 2, y = 3$$

$$2. \begin{cases} 2x + 3y = -1 \\ 3x + 4y = 0 \end{cases} \quad 3x = -4y \quad x = \frac{-4y}{3}$$

$$2 \cdot \left(\frac{-4y}{3} \right) + 3y = -1 \quad \frac{-8y}{3} + 3y = -1 \quad -8y + 9y = -3 \quad y = -3 \quad x = \frac{-4 \cdot (-3)}{3} \quad x = 4$$

$$3. \begin{cases} \frac{x + 3y}{2} = 5 \\ 3x - y = 5y \end{cases} \quad \begin{cases} x + 3y = 10 \\ 3x - y = 5y \end{cases} \quad \begin{cases} x = 10 - 3y \\ 3x = 6y \end{cases} \quad x = 2y$$

$$10 - 3y = 2y \quad y = 2 \quad x = 2 \cdot 2 \quad x = 4$$

$$4. \begin{cases} \frac{x + 3y}{2} = 5 \\ 4 - \frac{2x - y}{2} = 1 \end{cases} \quad \begin{cases} x + 3y = 10 \\ 8 - 2x + y = 2 \end{cases} \quad \begin{cases} x + 3y = 10 \\ -2x + y = -6 \end{cases} \quad \begin{cases} x = 10 - 3y \\ 7y - 14 \end{cases} \quad y = -2; x = 10 - 3 \cdot 2 \quad x = 4$$

$$5. \begin{cases} x + y = 60 \\ 16x + 20y = 1100 \end{cases}; x = 60 - y; 16(60 - y) + 20y = 1100 \quad 960 - 16y + 20y = 1100$$

$$4y = 140 \quad y = 35 \quad x + 35 = 60 \quad x = 25$$

$$6. \begin{cases} 3x + 2y = 7 \\ 4x - 3y = -2 \end{cases} \quad 3x - 7 - 2y \quad x = \frac{7 - 2y}{3}; 4 \left(\frac{7 - 2y}{3} \right) - 3y = -2; \quad \frac{28 - 8y}{3} - 3y = -2$$

$$28 - 8y - 9y = -6; \quad -17y = -34; \quad y = 2 \quad x = \frac{7 - 2 \cdot 2}{3} \quad x = 1$$

Ejercicio n° 2.

$$1. \begin{cases} 3x - 4y = -6 \\ 2x + 4y = 16 \end{cases} \quad 3x = -6 + 4y \quad x = \frac{-6 + 4y}{3}; \quad 2x = 16 - 4y \quad x = \frac{16 - 4y}{2}$$

$$\frac{-6 + 4y}{3} = \frac{16 - 4y}{2}; \quad 2(-6 + 4y) = 3(16 - 4y) \quad -12 + 8y = 48 - 12y$$

$$8y + 12y = 48 + 12 \quad 20y = 60 \quad y = 3$$

$$x = \frac{-6 + 4 \cdot 3}{3} = \frac{-6 + 12}{3} \quad x = 2 \quad x = 2, y = 3$$

$$2. \quad \begin{cases} 2x + 3y = -1 \\ 3x + 4y = 0 \end{cases} \quad 3x = -4y \quad x = \frac{-4y}{3}$$

$$2x = -1 - 3y \quad x = \frac{-1 - 3y}{2} \quad \frac{-1 - 3y}{2} = \frac{-4y}{3}$$

$$3(-1 - 3y) = 2(-4y) \quad -3 - 9y = -8y \quad y = -3 \quad x = \frac{-4 \cdot (-3)}{3} \quad x = 4$$

$$3. \quad \begin{cases} 3x + 2y = 7 \\ 4x - 3y = -2 \end{cases} \quad 3x = 7 - 2y \quad x = \frac{7 - 2y}{3}$$

$$4x = -2 + 3y \quad x = \frac{-2 + 3y}{4}; \quad 4(7 - 2y) = 3(-2 + 3y) \quad 28 - 8y = -6 + 9y$$

$$28 + 6 = 9y + 8y \quad 34 = 17y \quad y = 2 \quad x = \frac{7 - 2 \cdot 2}{3} \quad x = 1$$

$$4. \quad \begin{cases} \frac{x + 3y}{2} = 5 \\ 4 - \frac{2x - y}{2} = 1 \end{cases} \quad \begin{cases} x + 3y = 10 \\ 8 - 2x + y = 2 \end{cases} \quad \begin{cases} x = 10 - 3y \\ 3x = 6y \end{cases} \quad \begin{cases} x = 2y \\ x = 10 - 3y \end{cases}$$

$$10 - 3y = 2y \quad 10 = 5y \quad y = 2 \quad x = 10 - 3 \cdot 2 \quad x = 4$$

$$5. \quad \begin{cases} \frac{x + 3y}{2} = 5 \\ 4 - \frac{2x - y}{2} = 1 \end{cases} \quad \begin{cases} x + 3y = 10 \\ 8 - 2x + y = 2 \end{cases} \quad \begin{cases} x + 3y = 10 \\ -2x + y = -6 \end{cases} \quad \begin{cases} x = 10 - 3y \\ x = 10 - 3y \end{cases}$$

$$-2x = -6 - y \quad 2x = 6 + y \quad x = \frac{6 + y}{2}; \quad 10 - 3y = \frac{6 + y}{2}$$

$$20 - 6y = 6 + y \quad 14 = 7y \quad y = 2 \quad x = 10 - 3 \cdot 2 \quad x = 4$$

$$6. \quad \begin{cases} x + y = 60 \\ 16x + 20y = 1100 \end{cases}; \quad x = 60 - y; \quad 16x = 1100 - 20y \quad x = \frac{1100 - 20y}{16}$$

$$60 - y = \frac{1100 - 20y}{16} \quad 960 - 16y = 1100 - 20y \quad -16y + 20y = 1100 - 960$$

$$4y = 140 \quad y = 35 \quad x + 35 = 60 \quad x = 25$$

Ejercicio n° 3.-

$$1. \quad \begin{cases} 3x - 4y = -6 \\ 2x + 4y = 16 \end{cases} \quad \begin{cases} 3x - 4y = -6 \xrightarrow{\cdot 2} 6x - 8y = -12 \\ 2x + 4y = 16 \xrightarrow{\cdot (-3)} -6x - 12y = -48 \end{cases} \quad \begin{cases} 6x - 8y = -12 \\ -6x - 12y = -48 \\ -20y = -60 \end{cases} \quad \begin{cases} y = 3 \end{cases}$$

$$2x + 4 \cdot 3 = 16 \quad 2x + 12 = 16 \quad 2x = 4 \quad x = 2 \quad x = 2, y = 3$$

$$2. \begin{cases} 3x + 2y = 7 \\ 4x - 3y = -2 \end{cases} \begin{array}{l} \xrightarrow{\cdot 3} \\ \xrightarrow{\cdot 2} \end{array} \begin{cases} 9x + 6y = 21 \\ 8x - 6y = -4 \end{cases}$$

$$17x = 17 \quad x = 1$$

$$3 \cdot 1 + 2y = 7 \quad 2y = 4 \quad y = 2$$

$$3. \begin{cases} 3x + 2y = 24 \\ x + 3y = 3 \end{cases} \begin{array}{l} 3x + 2y = 24 \\ -3x - 9y = -9 \\ \hline -7y = 15 \end{array} \quad y = -\frac{15}{7}$$

$$x + 3\left(-\frac{15}{7}\right) = 3 \quad x - \frac{45}{7} = 3$$

$$7x - 45 = 21 \quad 7x = 66 \quad x = \frac{66}{7}$$

$$4. \begin{cases} x + y = 3500 \\ x - \frac{10x}{100} + y - \frac{8y}{100} = 3170 \end{cases}; \begin{cases} x + y = 3500 \\ 90x + 92y = 317000 \end{cases} \xrightarrow{\cdot (-90)} \begin{cases} -90x - 90y = -315000 \\ 90x + 92y = 317000 \\ \hline 2y = 2000 \end{cases}$$

$$y = 1000 \quad x + 1000 = 3500 \quad x = 2500$$

$$5. \begin{cases} 2x + 3y = -1 \\ 3x + 4y = 0 \end{cases} \quad 3x = -4y \quad x = \frac{-4y}{3}$$

$$2 \cdot \left(-\frac{4y}{3}\right) + 3y = -1 \quad \frac{-8y}{3} + 3y = -1 \quad -8y + 9y = -3 \quad y = -3; \quad x = \frac{-4 \cdot (-3)}{3} \quad x = 4$$

$$6. \begin{cases} \frac{x+y}{2} = x-1 \\ \frac{x-y}{2} = y+1 \end{cases} \quad \begin{cases} x+y = 2(x-1) \\ x-y = 2(y+1) \end{cases} \quad \begin{cases} x+y = 2x-2 \\ x-y = 2y+2 \end{cases}$$

$$\begin{cases} -x + y = -2 \\ x - 3y = 2 \end{cases} \begin{array}{l} -x + y = -2 \\ x - 3y = 2 \\ \hline -2y = 0 \end{array} \quad y = 0 \quad x - 3 \cdot 0 = 2 \quad x = 2$$

$$7. \begin{cases} x + y - 2000 \\ x + \frac{10x}{100} + y + \frac{15y}{100} = 2260 \end{cases}; \begin{cases} x + y = 2000 \\ 110x + 115y = 226000 \end{cases} \xrightarrow{\cdot (-110)} \begin{cases} -110x - 110y = -220000 \\ 110x + 115y = 226000 \\ \hline 5y = 6000 \end{cases}$$

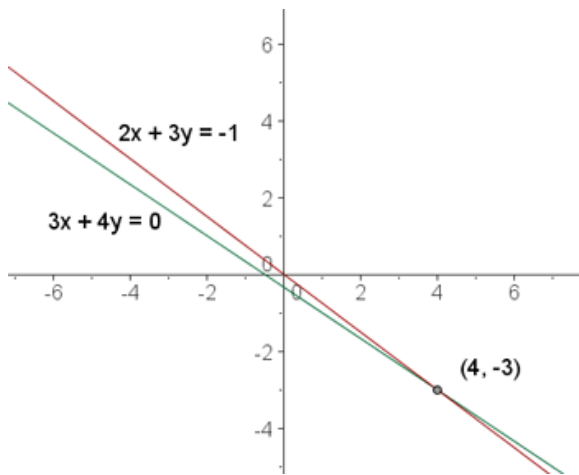
$$y = 1200 \quad x + 1200 = 2000 \quad x = 800$$

$$8. \begin{cases} x + y = 58 \\ 2x + 4y = 168 \end{cases} \quad \begin{cases} x + y = 58 \\ 2x + 4y = 168 \end{cases} \quad \begin{cases} -2x - 2y = -116 \\ 2x + 4y = 168 \\ \hline 2y = 52 \end{cases}$$

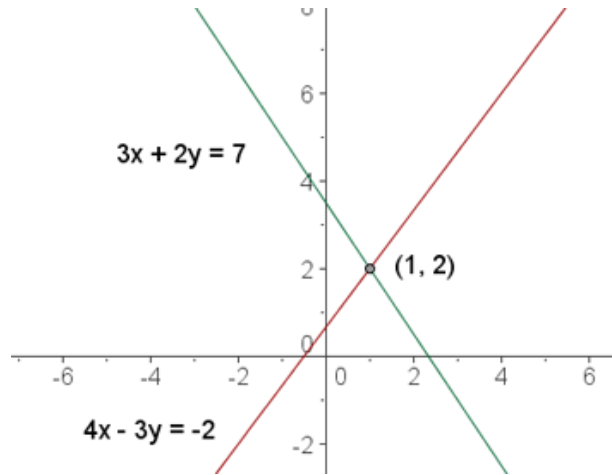
$$y = 26 \quad x + 26 = 58 \quad x = 32$$

Ejercicio nº 4.-

1a.



1b.



2a.
$$\begin{cases} \frac{x+y}{2} = x-1 \\ \frac{x-y}{2} = y+1 \end{cases} \quad \begin{cases} x+y = 2(x-1) \\ x-y = 2(y+1) \end{cases} \quad \begin{cases} x+y = 2x-2 \\ x-y = 2y+2 \end{cases}$$

$$\begin{cases} -x+y = -2 \\ x-3y = 2 \end{cases} \quad \begin{array}{r} -x+y = -2 \\ x-3y = 2 \\ \hline -2y = 0 \end{array} \quad y = 0 \quad x - 3 \cdot 0 = 2 \quad x = 2$$

2b.
$$\begin{cases} \frac{x+3y}{2} = 5 \\ 3x-y = 5y \end{cases} \quad \begin{cases} x+3y = 10 \\ 3x-y = 5y \end{cases} \quad \begin{array}{r} x+3y = 10 \\ 3x = 6y \\ \hline x = 10-3y \\ 3x = 6y \\ x = 2y \end{array}$$

$10 - 3y = 2y \quad y = 2 \quad x = 2 \cdot 2 \quad x = 4$

2c.
$$\begin{cases} \frac{x+3y}{2} = 5 \\ 4 - \frac{2x-y}{2} = 1 \end{cases} \quad \begin{cases} x+3y = 10 \\ 8-2x+y = 2 \end{cases} \quad \begin{cases} x+3y = 10 \\ -2x+y = -6 \end{cases} \quad \begin{array}{r} x+3y = 10 \\ -2x+y = -6 \\ \hline x = 10-3y \end{array}$$

$-2(10-3y) + y = -6 \quad -20 + 6y + y = -6 \quad 7y = 14 \quad y = 2; x = 10 - 3 \cdot 2 \quad x = 4$

2d.
$$\begin{cases} \frac{x}{2} + \frac{y}{3} = 4 \\ \frac{x}{3} + y = 1 \end{cases} \quad \begin{cases} 3x+2y = 24 \\ x+3y = 3 \end{cases} \quad \begin{array}{r} 3x+2y = 24 \\ -3x-9y = -9 \\ \hline -7y = 15 \end{array} \quad y = -\frac{15}{7}$$

$x + 3\left(-\frac{15}{7}\right) = 3 \quad x - \frac{45}{7} = 3$

$7x - 45 = 21 \quad 7x = 66 \quad x = \frac{66}{7}$

$$2e. \begin{cases} \frac{x+1}{3} + \frac{y-1}{2} = 0 \\ \frac{x+2y}{3} - \frac{x+y+2}{4} = 0 \end{cases} \quad \begin{cases} 2(x+1) + 3(y-1) = 0 \\ 4(x+2y) - 3(x+y+2) = 0 \end{cases}$$

$$\begin{cases} 2x+2+3y-3=0 \\ 4x+8y-3x-3y-6=0 \end{cases} \quad \begin{cases} 2x+3y=1 \\ x+5y=6 \end{cases} \quad \begin{array}{r} 2x+3y=1 \\ -2x-10y=-12 \\ \hline -7y=-11 \end{array} \quad y = -\frac{11}{7}$$

$$x + 5 \cdot \frac{11}{7} = 6 \quad x + \frac{55}{7} = 6 \quad 7x + 55 = 42$$

$$7x = -13 \quad x = -\frac{13}{7}$$

Ejercicio n° 5.-

$$1. \begin{cases} x^2 + y^2 = 25 \\ x + y = 7 \end{cases}; y=7-x; x^2 + (7-x)^2 = 25; x^2 + 49 - 14x + x^2 = 25; 2x^2 - 14x + 24 = 0; x^2 - 7x + 12 = 0$$

$$x = \frac{7 \pm \sqrt{49 - 48}}{2} = \frac{7 \pm 1}{2} \begin{array}{l} \nearrow x_1 = 4 \\ \searrow x_2 = 3 \end{array}; x = 3; y = 7 - 3; y = 4; x = 4; y = 7 - 4; y = 3$$

$$2. \begin{cases} x+y=7 \\ x \cdot y=12 \end{cases} \quad y = 7 - x \quad x \cdot (7 - x) = 12 \quad 7x - x^2 = 12 \quad x^2 - 7x + 12 = 0$$

$$x = \frac{7 \pm \sqrt{49 - 48}}{2} = \frac{7 \pm 1}{2} \begin{array}{l} \nearrow x_1 = 4 \\ \searrow x_2 = 3 \end{array}$$

$$x_1 = 4 \quad y = 7 - 4 \quad y_1 = 3 \quad x_2 = 3 \quad y = 7 - 3 \quad y_2 = 4$$

$$3. \begin{cases} x^2 + y^2 = 169 \\ x + y = 17 \end{cases} \quad x = 17 - y \quad 2y^2 - 34y + 120 = 0 \quad y^2 - 17y + 60 = 0$$

$$y = \frac{17 \pm \sqrt{289 - 240}}{2} = \frac{17 \pm 7}{2} \begin{array}{l} \nearrow y_1 = 12 \\ \searrow y_2 = 5 \end{array}$$

$$y_1 = 12 \quad x_1 = 17 - 12 \quad x_1 = 5 \quad y_2 = 5 \quad x_1 = 17 - 5 \quad x_2 = 12$$

$$4. \begin{cases} y^2 - 2y + 1 = x \\ \sqrt{x} + y = 5 \end{cases} \quad \sqrt{y^2 - 2y + 1} + y = 5 \quad (\sqrt{y^2 - 2y + 1})^2 = (5 - y)^2$$

$$y^2 - 2y + 1 = 25 - 10y + y^2 \quad 8y = 24 \quad y = 3$$

$$x = 3^2 - 2 \cdot 3 + 1 \quad x = 4$$

$$5. \begin{cases} \frac{1}{x^2} + \frac{1}{y^2} = 13 \\ \frac{1}{x} - \frac{1}{y} = 1 \end{cases} \quad \frac{1}{x} = u \quad \frac{1}{y} = v \quad \begin{cases} u^2 + v^2 = 13 \\ u - v = 1 \end{cases}$$

$$u = 1 + v \quad (1 + v)^2 + v^2 = 13 \quad 1 + 2v + v^2 + v^2 = 13 \quad v^2 + v - 6 = 0$$

$$v = \frac{-1 \pm \sqrt{1 + 24}}{2} = \frac{-1 \pm 5}{2} = \begin{cases} \nearrow v_1 = 2 & u_1 = 3 \\ \searrow v_2 = -3 & u_2 = -2 \end{cases}$$

$$\frac{1}{x_1} = 3 \quad x_1 = \frac{1}{3} \quad \frac{1}{x_2} = -2 \quad x_2 = -\frac{1}{2}$$

$$\frac{1}{y_1} = 2 \quad y_1 = \frac{1}{2} \quad \frac{1}{y_2} = -3 \quad y_2 = -\frac{1}{3}$$