MATEMATICAS. 4ºESO-B   TEMA 4: Ecuaciones

1.- Resuelve las siguientes **ecuaciones de 1er grado**:

1. $6\left(\dfrac{x+1}{8} - \dfrac{2x-3}{16}\right) - 3\left(\dfrac{3}{4}x - \dfrac{1}{4}\right) - \dfrac{3}{8}(3x-2)$ 2. $2 - \left[-2\cdot(x+1) - \dfrac{x-3}{2}\right] = \dfrac{2x}{3} - \dfrac{5x-3}{12} + 3x$

3. $\dfrac{2}{3}\left[x - \left(1 - \dfrac{x-2}{3}\right)\right] + 1 = x$ 4. $2 - \left[-2\cdot(x+1) - \dfrac{x-3}{2}\right] = \dfrac{2x}{3} - \dfrac{5x-3}{12} + 3x$

2.- Resuelve las siguientes **ecuaciones de 2º grado**:

1. $x^2 - 5x + 6 = 0$ 2. $2x^2 - 7x + 3 = 0$ 3. $-x^2 + 7x - 10 = 0$

4. $x^2 - 2x + 1 = 0$ 5. $x^2 + x + 1 = 0$ 6. $x^2 - 4x + 4 = 0$

7. $2x - 3 = 1 - 2x + x^2$ 8. $x^2 + (7-x)^2 = 25$ 9. $7x^2 + 21x - 28 = 0$

10. $-x^2 + 4x - 7 = 0$ 11. $18 = 6x + x(x-13)$ 12. $6x^2 - 5x + 1 = 0$

13. $x^2 + (x+2)^2 = 580$ 14. $x^2 - 5x - 84 = 0$ 15. $4x^2 - 6x + 2 = 0$

16. $x^2 - \dfrac{7}{6}x + \dfrac{1}{3} = 0$ 17. $\dfrac{2}{5}x^2 = 0$ 18. $x^2 - 5x = 0$

19. $2x^2 - 6x = 0$ 20. $x^2 - 25 = 0$ 21. $2x^2 + 8 = 0$

22. $12x^2 - 3x = 0$ 23. $4x^2 - 16 = 0$ 24. $6x^2 + 3x = 0$

3.- Resuelve las siguientes **ecuaciones bicuadradas**:

1. $x^4 - 10x^2 + 9 = 0$ 2. $x^4 - 13x^2 + 36 = 0$ 3. $x^4 - 61x^2 + 900 = 0$

4. $x^4 - 25x^2 + 144 = 0$ 5. $x^4 - 16x^2 - 225 = 0$ 6. $x^6 - 7x^3 + 6 = 0$

4.- Resuelve las siguientes **ecuaciones racionales**:

1. $\dfrac{1}{x^2-x} - \dfrac{1}{x-1} = 0$ 2. $\dfrac{1}{x-2} + \dfrac{1}{x+2} = \dfrac{1}{x^2-4}$ 3. $\dfrac{3}{x} = 1 + \dfrac{x-13}{6}$

5.- Resuelve las siguientes **ecuaciones irracionales**:

1. $\sqrt{2x-3} - x = -1$ 2. $\sqrt{5x+4} - 1 = 2x$ 3. $3\sqrt{x-1} + 11 = 2x$

4. $\sqrt{x} + \sqrt{x-4} = 2$ 5. $\sqrt{2x-1} + \sqrt{x+4} = 6$

6.- Resuelve las siguientes **ecuaciones polinómicas**:

1. $2x^4 + x^3 - 8x^2 - x + 6 = 0$ 2. $x^4 + 12x^3 - 64x^2 = 0$ 3. $2x^3 - 7x^2 + 8x - 3 = 0$

4. $x^3 - x^2 - 4 = 0$ 5. $6x^3 + 7x^2 - 9x + 2 = 0$ 6. $x^3 + 3x^2 - 4x - 12 = 0$

SOLUCIONES

Ejercicio nº 1.

1. $6\left(\dfrac{x+1}{8}-\dfrac{2x-3}{16}\right)=3\left(\dfrac{3}{4}x-\dfrac{1}{4}\right)-\dfrac{3}{8}(3x-2)$ $\qquad$ $\dfrac{6(x+1)}{8}-\dfrac{6(2x-3)}{16}=\dfrac{9}{4}x-\dfrac{3}{4}-\dfrac{9}{8}x+\dfrac{6}{8}$

$\dfrac{6x+6}{8}-\dfrac{12x-18}{16}=\dfrac{9}{4}x-\dfrac{3}{4}-\dfrac{9}{8}x+\dfrac{6}{8}$ $\qquad$ $mcm.(8, 16, 4)=16$

$2(6x+6)-(12x-18)=36x-12-18x+12$; $12x+12-12x+18=36x-12-18x+12$

$12+18=36x-18x$ $\qquad\qquad$ $18x=30$ $\qquad$ $3x=5$ $\qquad$ $x=\dfrac{5}{3}$

2. $2-\left[-2\cdot(x+1)-\dfrac{x-3}{2}\right]=\dfrac{2x}{3}-\dfrac{5x-3}{12}+3x$

$2-\left(-2x-2-\dfrac{x-3}{2}\right)=\dfrac{2x}{3}-\dfrac{5x-3}{12}+3x$

Quitamos corchete:

$2+2x+2+\dfrac{x-3}{2}=\dfrac{2x}{3}-\dfrac{5x-3}{12}+3x$

Quitamos paréntesis:

Quitamos denominadores: $\qquad 24+24x+24+6\cdot(x-3)=8x-(5x-3)+36x$

Quitamos paréntesis: $24+24x+24+6x-18=8x-5x+3+36x$

Agrupamos términos: $24x+6x-8x+5x-36x=3-24-24+18$

Sumamos: $\qquad -9x=-27$

Dividimos los dos miembros por: -9 $\qquad x=3$

3. $\dfrac{2}{3}\left[x-\left(1-\dfrac{x-2}{3}\right)\right]+1=x$ $\qquad\qquad$ $\dfrac{2}{3}\left(x-1+\dfrac{x-2}{3}\right)+1=x$

$\dfrac{2}{3}x-\dfrac{2}{3}+\dfrac{2x-4}{9}+1=x$ $\quad$ $6x-6+2x-4+9=9x$ $\qquad$ $-x=1$ $\qquad$ $x=-1$

4. $2-\left[-2\cdot(x+1)-\dfrac{x-3}{2}\right]=\dfrac{2x}{3}-\dfrac{5x-3}{12}+3x$; $2-\left(-2x-2-\dfrac{x-3}{2}\right)-\dfrac{2x}{3}-\dfrac{5x-3}{12}+3x$

$2+2x+2+\dfrac{x-3}{2}=\dfrac{2x}{3}-\dfrac{5x-3}{12}-3x$; $24+24x+24+6\cdot(x-3)-8x-(5x-3)+36x$

$24+24x+24+6x-18=8x-5x+3+36x$ $\qquad$ $-9x=-27$ $\qquad$ $x=3$

1. $$x^2 - 5x + 6 = 0$$

$$x = \frac{5 \pm \sqrt{5^2 - 4 \cdot 6}}{2} = \frac{5 \pm \sqrt{25 - 24}}{2} = \frac{5 \pm \sqrt{1}}{2} = \frac{5 \pm 1}{2} = \begin{cases} x_1 = \dfrac{6}{2} = 3 \\ x_2 = \dfrac{4}{2} = 2 \end{cases}$$

2. $$2x^2 - 7x + 3 = 0$$

$$x = \frac{7 \pm \sqrt{7^2 - 4 \cdot 2 \cdot 3}}{4} = \frac{7 \pm \sqrt{49 - 24}}{4} = \frac{7 \pm \sqrt{25}}{4} = \frac{7 \pm 5}{4} = \begin{cases} x_1 = \dfrac{12}{4} = 3 \\ x_2 = \dfrac{2}{4} = \dfrac{1}{2} \end{cases}$$

3. $$-x^2 + 7x - 10 = 0 \qquad (-1) \cdot (-x^2 + 7x - 10) = (-1) \cdot 0 \qquad x^2 - 7x + 10 = 0$$

$$x = \frac{7 \pm \sqrt{7^2 - 4 \cdot 10}}{2} = \frac{7 \pm \sqrt{49 - 40}}{2} = \frac{7 \pm \sqrt{9}}{2} = \frac{7 \pm 3}{2} = \begin{cases} x_1 = \dfrac{10}{2} = 5 \\ x_2 = \dfrac{4}{2} = 2 \end{cases}$$

4. $$x^2 - 2x + 1 = 0 \qquad x = \frac{2 \pm \sqrt{2^2 - 4 \cdot 1}}{2} = \frac{2 \pm \sqrt{4 - 4}}{2} = \frac{2 \pm \sqrt{0}}{2} = \frac{2 \pm 0}{2} = \frac{2}{2} = 1$$

5. $$x^2 + x + 1 = 0 \qquad x = \frac{-1 \pm \sqrt{1^2 - 4}}{2} = \frac{-1 \pm \sqrt{-3}}{2} \notin \mathbb{R}$$

6. $$x^2 - 4x + 4 = 0 \qquad x = \frac{4 \pm \sqrt{16 - 16}}{2} = \frac{4}{2} = 2$$

7. $$2x - 3 = 1 - 2x + x^2 \qquad x^2 - 4x + 4 = 0 \qquad x = \frac{4 \pm \sqrt{16 - 16}}{2} = \frac{4}{2} = 2$$

8. $$x^2 + (7 - x)^2 = 25 \qquad x^2 + 49 - 14x + x^2 = 25; \quad 2x^2 - 14x + 24 = 0; \quad x^2 - 7x + 12 = 0$$

$$x = \frac{7 \pm \sqrt{49 - 48}}{2} = \frac{7 \pm 1}{2} = \begin{cases} x_1 = 4 \\ x_2 = 3 \end{cases}$$

9. $$7x^2 + 21x - 28 = 0 \qquad x^2 + 3x - 4 = 0$$

$$x = \frac{-3 \pm \sqrt{9 + 16}}{2} = \frac{-3 \pm 5}{2} = \begin{cases} x_1 = 1 \\ x_2 = -4 \end{cases}$$

10. $-x^2 + 4x - 7 = 0$ $x^2 - 4x + 7 = 0$ $x = \dfrac{4 \pm \sqrt{16 - 28}}{2} = \dfrac{-3 \pm \sqrt{-12}}{2} \notin \mathbb{R}$

11. $18 = 6x + x(x - 13)$ $18 = 6x + x^2 - 13x$ $x^2 - 7x - 18 = 0$

$x = \dfrac{7 \pm \sqrt{49 + 72}}{2} = \dfrac{7 \pm \sqrt{121}}{2} = \dfrac{7 \pm 11}{2} = \;\nearrow\; x_1 = \dfrac{18}{2} = 9$
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \searrow\; x_2 = \dfrac{-4}{2} = -2$

12. $6x^2 - 5x + 1 = 0$

$x = \dfrac{5 \pm \sqrt{5^2 - 4 \cdot 6}}{12} = \dfrac{5 \pm \sqrt{25 - 24}}{12} = \dfrac{5 \pm \sqrt{1}}{12} = \dfrac{5 \pm 1}{12} = \;\nearrow\; x_1 = \dfrac{6}{12} = \dfrac{1}{2}$
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \searrow\; x_2 = \dfrac{4}{12} = \dfrac{1}{3}$

13. $x^2 + (x + 2)^2 = 580$ $x^2 + x^2 + 4x + 4 = 580$ $2x^2 + 4x - 576 = 0$

$x^2 + 2x - 288 = 0$

$x = \dfrac{-2 \pm \sqrt{4 + 1152}}{2} = \dfrac{-2 \pm 34}{2} = \;\nearrow\; x_1 = 16$
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad \searrow\; x_2 = -18$

14. $x^2 - 5x - 84 = 0$

$x = \dfrac{5 \pm \sqrt{25 + 336}}{2} = \dfrac{5 \pm \sqrt{361}}{2} = \dfrac{5 \pm 19}{2} = \;\nearrow\; x_1 = \dfrac{24}{2} = 12$
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \searrow\; x_2 = \dfrac{-14}{2} = -7$

15. $4x^2 - 6x + 2 = 0$

$x = \dfrac{5 \pm \sqrt{(-6)^2 - 4 \cdot 4 \cdot 2}}{8} = \dfrac{6 \pm \sqrt{36 - 32}}{8} = \dfrac{6 \pm \sqrt{4}}{8} = \dfrac{6 \pm 2}{8} = \;\nearrow\; x_1 = \dfrac{8}{8} = 1$
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \searrow\; x_2 = \dfrac{4}{8} = \dfrac{1}{2}$

16. $x^2 - \dfrac{7}{6}x + \dfrac{1}{3} = 0$; $6x^2 - 7x + 2 = 0$;

$x = \dfrac{7 \pm \sqrt{7^2 - 4 \cdot 6 \cdot 2}}{12} = \dfrac{7 \pm \sqrt{49 - 48}}{12} = \dfrac{7 \pm \sqrt{1}}{12} = \dfrac{7 \pm 1}{12} = \;\nearrow\; x_1 = \dfrac{8}{12} = \dfrac{2}{3}$
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \searrow\; x_2 = \dfrac{6}{12} = \dfrac{1}{2}$

17. $\dfrac{2}{5}x^2 = 0$ $2x^2 = 0$ $x = 0$

18. $x^2 - 5x = 0$ $x(x - 5) = 0$ $x = 0$ $x - 5 = 0$ $x = 5$

19. $2x^2 - 6x = 0$ $2x(x - 3) = 0$; $2x = 0$ $x = 0$ $x - 3 = 0$ $x = 3$

$$x^2 = 25 \qquad x = \pm\sqrt{25} \qquad \nearrow x_1 = \sqrt{25} = 5$$

20. $\quad x^2 - 25 = 0 \qquad\qquad\qquad\qquad \searrow x_2 = -\sqrt{25} = -5$

21. $\quad 2x^2 + 8 = 0 \qquad 2x^2 = -8 \qquad x^2 = -4 \qquad x = \pm\sqrt{-4} \notin \mathbb{R}$

22. $\quad 12x^2 - 3x = 0 \qquad 4x^2 - x = 0 \qquad x \cdot (4x - 1) = 0 \qquad$ **x = 0** $\quad 4x - 1 = 0 \quad$ **x = 1/4**

$$4x^2 = 16 \qquad x^2 = 4 \qquad x = \pm\sqrt{4} \quad \nearrow x_1 = 2$$

23. $\quad 4x^2 - 16 = 0 \qquad\qquad\qquad\qquad\qquad\qquad \searrow x_2 = -2$

24. $\quad 6x^2 + 3x = 0 \qquad 3x(2x + 1) = 0 \qquad 3x = 0 \qquad x = 0 ; \; \mathbf{2x + 1 = 0} \qquad x = -\dfrac{1}{2}$

Ejercicio nº 3.-

1. $x^4 - 10x^2 + 9 = 0 \qquad x^2 = t \qquad x^4 - 10x^2 + 9 = 0 \qquad t^2 - 10t + 9 = 0$

$$t = \frac{10 \pm \sqrt{10^2 - 4\cdot 9}}{2} = \frac{10 \pm \sqrt{100 - 36}}{2} = \frac{10 \pm \sqrt{64}}{2} = \frac{10 \pm 8}{2} = \quad \nearrow t_1 = \frac{18}{2} = 9$$
$$\searrow t_2 = \frac{2}{2} = 1$$

$$x^2 = 9 \qquad x = \pm\sqrt{9} = \pm 3 \qquad x^2 = 1 \qquad x = \pm\sqrt{1} = \pm 1$$

$$t = \frac{13 \pm \sqrt{169 - 144}}{2} = \frac{13 \pm 5}{2} = \quad \nearrow t_1 = \frac{18}{2} = 9$$
$$\searrow t_2 = \frac{8}{2} = 4$$

2. $x^4 - 13x^2 + 36 = 0 ; \; x^2 = t ; \; t^2 - 13t + 36 = 0$

$$x^2 = 9 \qquad x = \pm\sqrt{9} = \quad \nearrow x_1 = 3 \qquad\qquad x^2 = 4 \qquad x = \pm\sqrt{4} = \quad \nearrow x_3 = 2$$
$$\searrow x_2 = -3 \qquad\qquad\qquad\qquad\qquad\qquad \searrow x_4 = -2$$

$$t = \frac{61 \pm \sqrt{3721 - 3600}}{2} = \frac{61 \pm 11}{2} = \quad \nearrow t_1 = 36$$
$$\searrow t_2 = 25$$

3. $x^4 - 61x^2 + 900 = 0 ; \; x^2 = t ; \; t^2 - 61t + 900 = 0$

$$x^2 = 36 \qquad x = \pm\sqrt{36} \quad \nearrow x_1 = 6 \qquad\qquad x^2 = 25 \qquad x = \pm\sqrt{25} \quad \nearrow x_1 = 5$$
$$\searrow x_2 = -6 \qquad\qquad\qquad\qquad\qquad\qquad \searrow x_2 = -5$$

$$t = \frac{25 \pm \sqrt{625 - 576}}{2} = \frac{25 \pm 7}{2} = \quad \nearrow t_1 = 16$$
$$\searrow t_2 = 9$$

4. $x^4 - 25x^2 + 144 = 0 ; \; x^2 = t ; \; t^2 - 25t + 144 = 0$

$$x^2 = 16 \qquad x = \pm\sqrt{16} \quad \nearrow x_1 = 4 \qquad\qquad x^2 = 9 \qquad x = \pm\sqrt{9} \quad \nearrow x_3 = 3$$
$$\searrow x_2 = -4 \qquad\qquad\qquad\qquad\qquad\qquad \searrow x_4 = -3$$

5. $x^4 - 16x^2 - 225 = 0$; $x^2 = t$; $t^2 - 16t - 225 = 0$

$$t = \frac{16 \pm \sqrt{256 + 900}}{2} = \frac{16 \pm 34}{2} = \begin{cases} t_1 = 25 \\ t_2 = -9 \end{cases}$$

$x^2 = 25 \qquad x = \pm\sqrt{25} \nearrow \begin{array}{l} x_1 = 5 \\ x_2 = -5 \end{array} \quad x^2 = -9 \qquad x = \pm\sqrt{-9} \notin \mathbb{R}$

6. $x^6 - 7x^3 + 6 = 0$; $x^3 = t$; $t^2 - 7t + 6 = 0$

$$t = \frac{7 \pm \sqrt{49 - 24}}{2} = \frac{7 \pm 5}{2} = \begin{cases} t_1 = \dfrac{12}{2} = 6 \\ t_2 = \dfrac{2}{2} = 1 \end{cases}$$

$x^3 = 6 \qquad x = \sqrt[3]{6} \qquad x^3 = 1 \qquad x = \sqrt[3]{1} \qquad x = 1$

Ejercicio nº 4.-

1. $\dfrac{1}{x^2 - x} - \dfrac{1}{x - 1} = 0$ $\quad x^2 - x = x(x-1)$ $\quad m.c.m.(x^2 - x, x-1) = x(x-1)$; $1 - x = 0 \qquad x = 1$

Comprobamos la solución: $\dfrac{1}{1 - 1} - \dfrac{1}{1 - 1} = 0 \qquad \dfrac{1}{0} - \dfrac{1}{0} = 0$

La ecuación no tiene solución porque para x = 1 se anulan los denominadores.

2. $\dfrac{1}{x - 2} + \dfrac{1}{x + 2} = \dfrac{1}{x^2 - 4}$ $\qquad x^2 - 4 = (x-2)\cdot(x+2)$ mcm(x-2, x+2, x²-4) = (x-2) (x+2)

$x + 2 + x - 2 = 1 \quad 2x = 1 \qquad x = \dfrac{1}{2} \qquad \dfrac{1}{\frac{1}{2} - 2} + \dfrac{1}{\frac{1}{2} + 2} = \dfrac{1}{\left(\frac{1}{2}\right)^2 - 4}$

$\dfrac{1}{-\frac{3}{2}} + \dfrac{1}{\frac{5}{2}} = \dfrac{1}{\frac{-15}{4}} \qquad -\dfrac{2}{3} + \dfrac{2}{5} = -\dfrac{4}{15} \qquad -\dfrac{4}{15} = -\dfrac{4}{15} \qquad$ La solución es: $x = \dfrac{1}{2}$

3. $\dfrac{3}{x} = 1 + \dfrac{x - 13}{6}$; $m.c.m.(x, 6) = 6x$; $18 = 6x + x(x - 13)$; $18 = 6x + x^2 - 13x$

$$x = \frac{7 \pm \sqrt{49 + 72}}{2} = \frac{7 \pm \sqrt{121}}{2} = \frac{7 \pm 11}{2} = \begin{cases} x_1 = \dfrac{18}{2} = 9 \\ x_2 = \dfrac{-4}{2} = -2 \end{cases}$$

$x^2 - 7x - 18 = 0$

$\dfrac{3}{9} = 1 + \dfrac{9 - 13}{6} \qquad \dfrac{3}{9} = \dfrac{6 - 4}{6} \qquad \dfrac{3}{9} = \dfrac{2}{6} \qquad \dfrac{1}{3} = \dfrac{1}{3}$

$\dfrac{3}{-2} = 1 + \dfrac{-2 - 13}{6} \qquad \dfrac{3}{-2} = \dfrac{6 - 15}{6} \qquad \dfrac{3}{-2} = \dfrac{-9}{5} \qquad -\dfrac{3}{2} = -\dfrac{3}{2}$

1. $\sqrt{2x-3} = -1 + x$

1º Elevamos al cuadrado los dos miembros: $\left(\sqrt{2x-3}\right)^2 = (-1+x)^2$; $2x - 3 = 1 - 2x + x^2$

2º Resolvemos la ecuación: $x^2 - 4x + 4 = 0$ $\quad x = \dfrac{4 \pm \sqrt{16-16}}{2} = \dfrac{4}{2} = 2$

3º Comprobamos: $\sqrt{2 \cdot 2 - 3} - 2 = -1$ $\quad 1 - 2 = -1$; La solución es x = 2.

2. $\sqrt{5x+4} - 1 = 2x$ $\qquad \sqrt{5x+4} = 2x + 1$ $\qquad \left(\sqrt{5x+4}\right)^2 = (2x+1)^2$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad x = \dfrac{1 \pm \sqrt{1+48}}{8} = \dfrac{1 \pm 7}{8} = \begin{cases} x_1 = \dfrac{8}{8} - 1 \\[2mm] x_2 = \dfrac{-6}{8} = -\dfrac{3}{4} \end{cases}$

$5x + 4 = 4x^2 + 4x + 1$ $\qquad 4x^2 - x - 3 = 0$

$\sqrt{5 \cdot 1 + 4} - 1 = 2 \cdot 1$ $\qquad 3 - 1 = 2$ $\qquad x = 1$; $\sqrt{5 \cdot \left(-\dfrac{3}{4}\right) + 4} - 1 = 2 \cdot \left(-\dfrac{3}{4}\right)$ $\quad -\dfrac{1}{2} \neq -\dfrac{3}{2}$

3. $3\sqrt{x-1} + 11 = 2x$ $\qquad 3\sqrt{x-1} = 2x - 11$ $\qquad \left(3\sqrt{x-1}\right)^2 = (2x-11)^2$

$9(x-1) = 4x^2 - 44x + 121$ $\qquad 9x - 9 = 4x^2 - 44x + 121$ $\qquad 4x^2 - 53x + 130 = 0$

$x = \dfrac{53 \pm \sqrt{2809 - 2080}}{8} = \dfrac{53 \pm \sqrt{729}}{8} = \dfrac{53 \pm 27}{8} = \begin{cases} x_1 = \dfrac{80}{8} = 10 \\[2mm] x_2 = \dfrac{26}{8} = \dfrac{13}{4} \end{cases}$

$3\sqrt{10-1} + 11 = 2 \cdot 10$ $\qquad 20 = 20$ $\qquad 3\sqrt{\dfrac{13}{4} - 1} + 11 \neq 2 \cdot \dfrac{13}{4}$

4. $\sqrt{x} + \sqrt{x-4} = 2$ $\qquad \sqrt{x} = 2 - \sqrt{x-4}$ $\qquad \left(\sqrt{x}\right)^2 = \left(2 - \sqrt{x-4}\right)^2$

$x = 4 - 4\sqrt{x-4} + x - 4$ $\qquad 4\sqrt{x-4} = 0$ $\qquad \sqrt{x-4} = 0$

$\left(\sqrt{x-4}\right)^2 = 0^2$ $\qquad x - 4 = 0$ $\qquad x = 4$

$\sqrt{4} + \sqrt{4-4} = 2$ $\qquad 2 + 0 = 2$

La ecuación tiene por solución x = 4.

5. $\sqrt{2x-1} + \sqrt{x+4} = 6$ $\qquad$ $\sqrt{2x-1} = 6 - \sqrt{x+4}$ $\qquad$ $\left(\sqrt{2x-1}\right)^2 = \left(6 - \sqrt{x+4}\right)^2$

$2x-1 = 36 - 12\sqrt{x+4} + x + 4$; $x - 41 = -12\sqrt{x+4}$; $(x-41)^2 = \left(-12\sqrt{x+4}\right)^2$

$$x^2 - 226x + 1105 = 0 \nearrow \begin{matrix} x_1 = 5 \\ \\ x_2 = 221 \end{matrix}$$

$x^2 - 82x + 1681 = 144x + 576$

$\sqrt{2\cdot5-1} + \sqrt{5+4} = 6$ $\qquad$ $3 + 3 = 6$ $\qquad$ $x = 5$; $\sqrt{2\cdot221-1} + \sqrt{221+4} = 6$ $\qquad$ $21 + 15 \neq 6$

Ejercicio nº 6.-

1. $2x^4 + x^3 - 8x^2 - x + 6 = 0$ $\qquad$ $P(1) = 2\cdot1^4 + 1^3 - 8\cdot1^2 - 1 + 6 = 2 + 1 - 8 - 1 + 6 = 0$

$$\begin{array}{c|ccccc} & 2 & 1 & -8 & -1 & 6 \\ 1 & & 2 & 3 & -5 & -6 \\ \hline & 2 & 3 & -5 & -6 & 0 \end{array}$$

$(x-1)\cdot(2x^3 + 3x^2 - 5x - 6) = 0$

$P(1) = 2\cdot1^3 + 3\cdot1^2 - 5x - 6 \neq 0$ $\qquad$ $P(-1) = 2\cdot(-1)^3 + 3\cdot(-1)^2 - 5\cdot(-1) - 6 = -2 + 3 + 5 - 6 = 0$

$$\begin{array}{c|cccc} & 2 & 3 & -5 & -6 \\ -1 & & -2 & -1 & 6 \\ \hline & 2 & 1 & -6 & 0 \end{array}$$

$(x-1)\cdot(x+1)\cdot(2x^2 + x - 6) = 0$

$2x^2 + x - 6 = 0$; Las soluciones son: $x = 1$, $x = -1$, $x = -2$ y $x = 3/2$

2. $x^4 + 12x^3 - 64x^2 = 0$ $\qquad$ $x^2(x^2 + 12x - 64) = 0$ $\quad$ $x^2 = 0$ $\qquad$ $x_1 = 0$

$$x = \frac{-12 \pm \sqrt{144 + 256}}{2} = \frac{-12 \pm 20}{2} = \nearrow \begin{matrix} x_2 = 4 \\ \\ x_3 = -16 \end{matrix}$$

$x^2 + 12x - 64 = 0$

3. $2x^3 - 7x^2 + 8x - 3 = 0$ $\qquad$ $P(1) = 2\cdot1^3 - 7\cdot1^2 + 8\cdot1 - 3 = 0$

$$\begin{array}{c|cccc} & 2 & -7 & 8 & -3 \\ 1 & & 2 & -5 & 3 \\ \hline & 2 & -5 & 3 & 0 \end{array}$$

$(x-1)\cdot(2x^2 - 5x + 3) = 0$ $\quad$ $P(1) = 2\cdot1^2 - 5\cdot1 + 3 = 0$

$$\begin{array}{c|ccc} & 2 & -5 & 3 \\ 1 & & 2 & -3 \\ \hline & 2 & -3 & 0 \end{array}$$

$(x-1)^2\cdot(2x-3) = 0$

Las raíces son x = 3/2 y x = 1

4. $x^3 - x^2 - 4 = 0$ Posibles raíces enteras: $\{\pm 1, \pm 2, \pm 4\}$

$P(1) = 1^3 - 1^2 - 4 \neq 0$ 　　　 $P(-1) = (-1)^3 - (-1)^2 - 4 \neq 0$ 　　　 $P(2) = 2^3 - 2^2 - 4 = 8 - 4 - 4 = 0$

$$
\begin{array}{c|cccc}
 & 1 & -1 & 0 & -4 \\
2 & & 2 & 2 & 4 \\
\hline
 & 1 & 1 & 2 & 0 \\
\end{array}
$$

$(x - 2) \cdot (x^2 + x + 2) = 0$ 　　　 $x^2 + x + 2 = 0$

$x = \dfrac{-1 + \sqrt{(-1)^2 - 4 \cdot 2}}{2} = \dfrac{-1 \pm \sqrt{1 - 8}}{2} = \dfrac{-1 \pm \sqrt{-7}}{2} \notin \mathbb{R}$ 　　　 $(x - 2) \cdot (x^2 + x + 2) = 0$ 　　 Raíz: $x = 2$.

5. $6x^3 + 7x^2 - 9x + 2 = 0$ 　　　 Posibles raíces enteras: $\{\pm 1, \pm 2\}$

$P(1) = 6 \cdot 1^3 + 7 \cdot 1^2 - 9 \cdot 1 + 2 \neq 0$ 　　　　 $P(-1) = 6 \cdot (-1)^3 + 7 \cdot (-1)^2 - 9 \cdot (-1) + 2 \neq 0$

$P(2) = 6 \cdot 2^3 + 7 \cdot 2^2 - 9 \cdot 2 + 2 \neq 0$ 　　　　 $P(-2) = 6 \cdot (-2)^3 + 7 \cdot (-2)^2 - 9 \cdot (-2) + 2 = -48 + 28 + 18 + 2 = 0$

$$
\begin{array}{c|cccc}
 & 6 & 7 & -9 & 2 \\
-2 & & -12 & 10 & -2 \\
\hline
 & 6 & -5 & 1 & 0 \\
\end{array}
$$

$(x + 2) \cdot (6x^2 - 5x + 1) = 0$ 　　　 $6x^2 - 5x + 1 = 0$

$x = \dfrac{5 \pm \sqrt{5^2 - 4 \cdot 6}}{12} = \dfrac{5 \pm \sqrt{25 - 24}}{12} = \dfrac{5 \pm \sqrt{1}}{12} = \dfrac{5 \pm 1}{12} = \begin{cases} x_1 = \dfrac{6}{12} = \dfrac{1}{2} \\[2mm] x_2 = \dfrac{4}{12} = \dfrac{1}{3} \end{cases}$

$6 (x + 2) \cdot (x - 1/2) \cdot (x - 1/3) = 0$ 　　　 **Raíces: $x = -2$, $x = 1/2$ y $x = 1/3$**

6. $x^3 + 3x^2 - 4x - 12 = 0$ 　　　　　 Posibles raíces enteras: $\{\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12\}$

$P(1) = 1^3 + 3 \cdot 1^2 - 4 \cdot 1 - 12 \neq 0$ 　　 $P(-1) = (-1)^3 + 3 \cdot (-1)^2 - 4 \cdot (-1) - 12 \neq 0$

$P(2) = 2^3 + 3 \cdot 2^2 - 4 \cdot 2 - 12 = 8 + 12 - 8 - 12 = 0$

$$
\begin{array}{c|cccc}
 & 1 & 3 & -4 & -12 \\
2 & & 2 & 10 & 12 \\
\hline
 & 1 & 5 & 6 & 0 \\
\end{array}
$$

$(x - 2) \cdot (x^2 - 5x + 6) = 0$ 　　 $x^2 - 5x + 6 = 0$

$x = \dfrac{-5 \pm \sqrt{5^2 - 4 \cdot 5}}{2} = \dfrac{-5 \pm \sqrt{25 - 24}}{2} = \dfrac{-5 \pm \sqrt{1}}{2} = \dfrac{-5 \pm 1}{2} = \begin{cases} x_1 = \dfrac{-4}{2} = -2 \\[2mm] x_2 = \dfrac{-5}{2} = -3 \end{cases}$

$(x - 2) \cdot (x + 2) \cdot (x + 3) = 0$ 　　　 **Las soluciones son : $x = 2$, $x = -2$, $x = -3$**