MATEMATICAS. BC2 TEMA 11: Integrales inmediatas-II

1. Resolver las siguientes **integrales inmediatas:**

1 $\int \cos x \sqrt{sen\, x}\, dx$

2 $\int \sec^3 x\, tg\, x\, dx$

3 $\int \frac{dx}{\sqrt{x} \cos^2 \sqrt{x}}$

4 $\int \frac{dx}{x(1+\ln x)^3}$

5 $\int \frac{sen^2 x}{\cos^4 x} dx$

6 $\int \frac{sen\, x + tg\, x}{\cos x} dx$

7 $\int \frac{dx}{sen\, x \cos x}$

8 $\int \sqrt{x\sqrt{x}}\, dx$

9 $\int \sqrt[3]{x\sqrt{\frac{2}{x}}}\, dx$

10 $\int \frac{\cos x}{\sqrt{sen^3 x}} dx$

11 $\int \frac{sen\, x - \cos x}{sen\, x + \cos x} dx$

12 $\int \frac{dx}{(1+x^2) arc\, tg\, x}$

13 $\int \frac{\sec^2 x}{1 + tg^2 x} dx$

14 $\int \frac{(2\ln x)^2}{4x} dx$

15 $\int \frac{\ln x^2}{x} dx$

16 $\int \frac{dx}{\sqrt{x} \cos^2 \sqrt{x}}$

17 $\int \frac{4^x + 5\cdot 16^x}{1+16^x} dx$

18 $\int \frac{tg \sqrt{x}}{\sqrt{x}} dx$

19 $\int \frac{\sqrt{7+2\, tg\, x}}{\cos^2 x} dx$

20 $\int \frac{dx}{x\sqrt{1-\ln^2 x}}$

2. De las infinitas funciones primitivas de la función $y= x^2-x+1$, ¿cuál es la que para $x = 3$ toma el valor 5?

3. Escribe la función primitiva de $y = x^2 + 2x$ cuya representación gráfica pasa por él punto (1, 3).

4. Hallar una función F(x) cuya derivada sea $f(x) = x + 6$ y tal que para $x = 2$ tome el valor 25.

5. Hallar la recta de pendiente 2 que pasa por el punto P(0, 4).

6. Calcular la ecuación de la curva que pasa por P(1, 5) y cuya pendiente en cualquier punto es $3x^2 + 5x - 2$.

7. Hallar la primitiva de la función $f(x) = x\sqrt{x^2 - 1}$, que se anula para $x = 2$

SOLUCIONES

Ejercicio 1:

1 $\int \cos x\sqrt{sen\,x}\,dx = \int (sen\,x)^{\frac{1}{2}}\cos x\,dx = \dfrac{(sen\,x)^{\frac{3}{2}}}{\frac{3}{2}} + C = \; = \dfrac{2}{3}sen\,x\sqrt{sen\,x} + C$

2 $y = \sec x \qquad y' = \sec x\,tg\,x\,; \quad \int \sec^2 x \sec x\,tg\,x\,dx = \dfrac{1}{3}\sec^3 x + C$

3 $\int \dfrac{dx}{\sqrt{x}\cos^2\sqrt{x}} = 2\int \dfrac{\frac{1}{2\sqrt{x}}}{\sqrt{x}\cos^2\sqrt{x}}dx = 2\,tg\sqrt{x} + C$

4 $\int \dfrac{dx}{x(1+\ln x)^3} = \int (1+\ln x)^{-3}\dfrac{1}{x}dx = -\dfrac{(1+\ln x)^{-2}}{2} + C = = -\dfrac{1}{2(1+\ln x)^2} + C$

5 $\int \dfrac{sen^2 x}{\cos^4 x}dx = \int \dfrac{sen^2 x}{\cos^2 x}\,\dfrac{1}{\cos^2 x}dx = \int tg^2 x\,\dfrac{1}{\cos^2 x}dx = = \dfrac{1}{3}tg^3 x + C$

6 $\int \dfrac{sen\,x + tg\,x}{\cos x}dx = \int \dfrac{sen\,x}{\cos x}dx + \int \dfrac{tg\,x}{\cos x}dx = = \int \dfrac{sen\,x}{\cos x}dx + \int tg\,x\sec x\,dx = -\ln(\cos x) + \sec x + C$

7 $\int \dfrac{dx}{sen\,x\cos x} = \int \dfrac{sen^2 x + \cos^2 x}{sen\,x\cos x}dx = \int \dfrac{sen\,x}{\cos x}dx + \int \dfrac{\cos x}{sen\,x}dx = = -\ln(\cos x) + \ln(sen\,x) + C = \ln(tg\,x) + C$

8 $\int \sqrt{x\sqrt{x}}\,dx \int \sqrt{\sqrt{x^2\cdot x}}\,dx = \int \sqrt[4]{x^3}\,dx = \int x^{\frac{3}{4}}dx = \int \dfrac{x^{\frac{3}{4}+1}}{\frac{3}{4}+1}dx = = \dfrac{4}{7}x^{\frac{7}{4}} + C = \dfrac{4}{7}\sqrt[4]{x^7} + C = \dfrac{4}{7}x\sqrt[4]{x^3} + C$

9 $\int \sqrt[3]{x\sqrt{\dfrac{2}{x}}}\,dx = \int \sqrt[3]{\sqrt{\dfrac{2x^2}{x}}}\,dx = \int \sqrt[6]{2x}\,dx = \sqrt[6]{2}\int x^{\frac{1}{6}}dx = \sqrt[6]{2}\int \dfrac{x^{\frac{1}{6}+1}}{\frac{1}{6}+1}dx =$

$= \dfrac{6}{7}\sqrt[6]{2}\,x^{\frac{7}{6}} + C = \dfrac{6}{7}\sqrt[6]{2}\,\sqrt[6]{x^7} + C = \dfrac{6}{7}x\sqrt[6]{2x} + C$

10 $\int \dfrac{\cos x}{\sqrt{sen^3 x}}dx = \int \cos x\,sen^{\frac{-3}{2}}x\,dx = \dfrac{sen^{\frac{-1}{2}}x}{\frac{-1}{2}} + C = \dfrac{-2}{\sqrt{sen\,x}} + C$

11 $\int \dfrac{sen\,x - \cos x}{sen\,x + \cos x}dx = -\int \dfrac{\cos x - sen\,x}{sen\,x + \cos x}dx = = -\ln(sen\,x + \cos x) + C$

12 $\int \dfrac{dx}{(1+x^2)\,arc\,tg\,x} = \int \dfrac{\frac{1}{1+x^2}}{arc\,tg\,x}dx = = \ln(arc\,tg\,x) + C$

13 $\int \dfrac{\sec^2 x}{1+tg^2 x}dx = arc\,tg(tg\,x) + C = x + C$

14 $\int \dfrac{(2\ln x)^2}{4x}dx = \int \ln^2(x)\,\dfrac{1}{x}dx = \dfrac{1}{3}\ln^3 x + C$

15 $\int \dfrac{\ln x^2}{x}dx = \int 2\ln(x)\,\dfrac{1}{x}dx = \ln^2 x + C$

16 $\int \frac{dx}{\sqrt{x}\cos^2\sqrt{x}} = 2\int \frac{dx}{\cos^2\sqrt{x}}\ \frac{1}{2\sqrt{x}}\ dx = 2\ \mathrm{tg}\sqrt{x}\ + C$

17 $\int \frac{4^x + 5\cdot 16^x}{1+16^x}dx = \int \frac{4^x}{1+(4^x)^2}dx + 5\int \frac{4^{2x}}{1+4^{2x}}dx = = \frac{1}{\ln 4}\mathrm{arc\ tg}(4^x) + \frac{5}{2\ln 4}\ln(1+4^{2x})\ + C$

18 $-2\int \frac{-\mathrm{sen}\sqrt{x}}{\cos\sqrt{x}}\ \frac{1}{2\sqrt{x}}\ dx = -2\ln(\cos\sqrt{x})\ + C$

19 $\int \frac{\sqrt{7+2\,\mathrm{tg}\,x}}{\cos^2 x}dx = \frac{1}{2}\int (7+2\,\mathrm{tg}\,x)^{\frac{1}{2}}\frac{2}{\cos^2 x}dx = = \frac{1}{2}\frac{(7+2\,\mathrm{tg}\,x)^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{1}{3}(7+2\,\mathrm{tg}\,x)\sqrt{(7+2\,\mathrm{tg}\,x)} + C$

20 $\int \frac{dx}{x\sqrt{1-\ln^2 x}} = \int \frac{\frac{1}{x}}{\sqrt{1-\ln^2 x}}dx = \mathrm{arc\ sen}(\ln x)\ + C$

Ejercicio 2:

$\int (x^2 - x + 1)dx = \frac{x^3}{3} - \frac{x^2}{2} + x + C$; $y = \frac{x^3}{3} - \frac{x^2}{2} + x + C$; x = 3 y = 5

$5 = \frac{3^3}{3} - \frac{3^2}{2} + 3 + C$; $C = -\frac{5}{2}$; $y = \frac{x^3}{3} - \frac{x^2}{2} + x - \frac{5}{2}$

Ejercicio 3:

$\int (x^2 + 2x)dx = \frac{x^3}{3} + \frac{2x^2}{2} + C = \frac{x^3}{3} + x^2 + C$; $y = \frac{x^3}{3} + x^2 + C$

$3 = \frac{1^3}{3} + 1^2 + C$ $\quad C = \frac{5}{3}$; $y = \frac{x^3}{3} + x^2 + \frac{5}{3}$

Ejercicio 4:

$\int (x+6)\,dx = \frac{x^2}{2} + 6x\ +\ C$; $25 = \frac{2^2}{2} + 6\cdot 2\ +\ C$ $\quad C = 11$; $F(x) = \frac{1}{2}x^2 + 6x + 11$

Ejercicio 5:

f '(x) = 2; f(x) = 2x + C; 4 = 2 · 0 + C; f(x) = 2x + 4

Ejercicio 6:

$y' = 3x^2 + 5x - 2$; $y = \int (3x^2 + 5x - 2)dx = x^3 + \frac{5}{2}x^2 - 2x + C$

$5 = 1 + \frac{5}{2} - 2 + C$ $\quad C = \frac{7}{2}$; $y = x^3 + \frac{5}{2}x^2 - 2x + \frac{7}{2}$

Ejercicio 7:

$\int x\sqrt{x^2 - 1}\ dx = \frac{1}{2}\int 2x\ (x^2-1)^{\frac{1}{2}}\ dx = \frac{1}{2}\frac{(x^2-1)^{\frac{3}{2}}}{\frac{3}{2}}\ +\ C$

$\frac{1}{3}\sqrt{(2^2-1)^3}\ +\ C = 0$ $\quad C = -\sqrt{3}$ $\quad F(x) = \frac{1}{3}\sqrt{(x^2-1)^3} - \sqrt{3}$