MATEMATICAS. BC2 TEMA 8: Derivadas

1. Calcular aplicando la definición las derivadas en los puntos que se indica:

1 $f(x)=2x^2-6x+5$ en x = -5 2 $f(x)=x^3+2x-5$ en x = 1

3 $f(x)=\dfrac{1}{x}$ en x = 2 4 $f(x)=\sqrt{x}$ en x = 3.

2. Calcula las derivadas de las funciones:

1 $f(x)=5$ 2 $f(x)=-2x$

3 $f(x)=-2x+2$ 4 $f(x)=-2x^2-5$

5 $f(x)=2x^4+x^3-x^2+4$ 6 $f(x)=\dfrac{x^3+2}{3}$

7 $f(x)=\dfrac{1}{3x^2}$ 8 $f(x)=\dfrac{x+1}{x-1}$ 9 $f(x)=\left(5x^2-3\right)\cdot\left(x^2+x+4\right)$

3. Calcula mediante la fórmula de la derivada de una potencia:

1 $f(x)=\dfrac{5}{x^5}$ 2 $f(x)=\dfrac{5}{x^5}+\dfrac{3}{x^2}$

3 $f(x)=\sqrt{x}$ 4 $f(x)=\dfrac{1}{\sqrt{x}}$

5 $f(x)=\dfrac{1}{x\sqrt{x}}$ 6 $f(x)=\sqrt[3]{x^2}+\sqrt{x}$ 7 $f(x)=\left(x^2+3x-2\right)^4$

4. Calcula mediante la fórmula de la derivada de una raíz:

1 $f(x)=\sqrt{x^2-2x+3}$ 2 $f(x)=\sqrt[4]{x^5-x^3-2}$ 3 $f(x)=\sqrt[3]{\dfrac{x^2+1}{x^2-1}}$

5. Deriva las funciones exponenciales

1 $f(x)=10^{\sqrt{x}}$ 2 $f(x)=e^{3-x^2}$

3 $f(x)=\dfrac{e^x+e^{-x}}{2}$ 4 $f(x)=3^{2x^2}\cdot\sqrt{x}$ 5 $f(x)=\dfrac{e^{2x}}{x^2}$

6. Calcula la derivada de las funciones logarítmicas:

1 $f(x)=\ln\left(2x^4-x^3+3x^2-3x\right)$ 2 $f(x)=\ln\left(\dfrac{e^x+1}{e^x-1}\right)$

3 $f(x)=\log\sqrt{\dfrac{1+x}{1-x}}$ 4 $f(x)=\ln\sqrt{x\,(1-x)}$ 5 $f(x)=\ln\sqrt[3]{\dfrac{3x}{x+2}}$

SOLUCIONES

Ejercicio 1:

1 $$f'(x)=\lim_{h\to 0}\frac{2(x+h)^2-6(x+h)+5-(2x^2-6x+5)}{h}==\lim_{h\to 0}\frac{2x^2+4xh+2h^2-6x-6h+5-2x^2+6x-5}{h}=$$

$$=\lim_{h\to 0}\frac{4xh-6h+2h^2}{h}=\lim_{h\to 0}\frac{h(4x-6+2h)}{h}=4x-6 \qquad f'(-5)=4(-5)-6=-26$$

2 $$f'(x)=\lim_{h\to 0}\frac{(x+h)^3+2(x+h)-5-(x^3+2x-5)}{h}==\lim_{h\to 0}\frac{x^3+3x^2h+3xh^2+h^3+2x+2h-5-x^3-2x+5}{h}=$$

$$=\lim_{h\to 0}\frac{3x^2h+3xh^2+h^3+2h}{h}=\lim_{h\to 0}\frac{h(3x^2+3xh+h^2+2)}{h}=3x^2+2 \qquad f'(1)=3(1)^2+2=5$$

3 $$f'(x)=\lim_{h\to 0}\frac{\frac{1}{x+h}-\frac{1}{x}}{h}=\lim_{h\to 0}\frac{\frac{x-(x+h)}{x(x+h)}}{h}=\lim_{h\to 0}\frac{\frac{-h}{x^2+xh}}{h}==\lim_{h\to 0}\left(-\frac{1}{x^2+xh}\right)=-\frac{1}{x^2} \quad f'(2)=-\frac{1}{2^2}=-\frac{1}{4}$$

4 $$f'(3)=\lim_{h\to 0}\frac{\sqrt{3+h}-\sqrt{3}}{h}=\lim_{h\to 0}\frac{(\sqrt{3+h}-\sqrt{3})(\sqrt{3+h}+\sqrt{3})}{h(\sqrt{3+h}+\sqrt{3})}=$$

$$\lim_{h\to 0}\frac{(3+h)-3}{h(\sqrt{3+h}+\sqrt{3})}=\lim_{h\to 0}\frac{h}{h(\sqrt{3+h}+\sqrt{3})}= \quad \lim_{h\to 0}\frac{1}{\sqrt{3+h}+\sqrt{3}}=\frac{1}{\sqrt{3}+\sqrt{3}}=\frac{1}{2\sqrt{3}}$$

Ejercicio 2:

1 $f(x)=5 \qquad f'(x)=0$

2 $f(x)=-2x \qquad f'(x)=-2$

3 $f(x)=-2x+2 \qquad f'(x)=-2$

4 $f(x)=-2x^2-5 \qquad f'(x)=-4x$

5 $f(x)=2x^4+x^3-x^2+4 \quad f'(x)=8x^3+3x^2-2x$

6 $f(x)=\dfrac{x^3+2}{3} \qquad f'(x)=x^2$

7 $f(x)=\dfrac{1}{3x^2} \quad f'(x)=\dfrac{-6x}{(3x)^2}=\dfrac{-6x}{9x^4}=-\dfrac{2}{3x^3}$

8 $$f(x)=\frac{x+1}{x-1} \qquad f'(x)=\frac{1\cdot(x-1)-(x+1)\cdot 1}{(x-1)^2}=\frac{-2}{(x-1)^2}$$

9 $f(x)-(5x^2-3)\cdot(x^2+x+4)$; $f'(x)=10x(x^2+x+4)+(5x^2-3)(2x+1)=20x^3+15x^2+34x-3$

Ejercicio 3:

1 $$f(x)=\frac{5}{x^5}=5x^{-5} \qquad f'(x)=-25x^{-6}=-\frac{25}{x^6}$$

2 $$f(x)=\frac{5}{x^5}+\frac{3}{x^2}=5x^{-5}+3x^{-2} \qquad f'(x)=-25x^{-6}-6x^{-3}=-\frac{25}{x^6}-\frac{6}{x^3}$$

3 $$f(x)=\sqrt{x}=x^{\frac{1}{2}} \qquad f'(x)=\frac{1}{2}x^{\frac{1}{2}-1}=\frac{1}{2}x^{-\frac{1}{2}}=\frac{1}{2\sqrt{x}}$$

4 $$f(x)=\frac{1}{\sqrt{x}}=x^{-\frac{1}{2}} \qquad f'(x)=-\frac{1}{2}x^{-\frac{1}{2}-1}=-\frac{1}{2}x^{-\frac{3}{2}}=-\frac{1}{2\sqrt{x^3}}$$

5 $f(x)=\frac{1}{x\sqrt{x}}=\frac{1}{x\ x^{\frac{1}{2}}}=x^{\frac{-3}{2}}$ $\quad f'(x)=\frac{-3}{2}x^{\frac{-5}{2}}=-\frac{3}{2\sqrt{x^5}}$

6 $f(x)=\sqrt[3]{x^2}+\sqrt{x}=x^{\frac{2}{3}}+x^{\frac{1}{2}}$; $\quad f'(x)=\frac{2}{3}x^{\frac{2}{3}-1}+\frac{1}{2}x^{\frac{1}{2}-1}=\frac{2}{3}x^{-\frac{1}{3}}+\frac{1}{2}x^{-\frac{1}{2}}=\frac{2}{3\sqrt[3]{x}}+\frac{1}{2\sqrt{x}}$

7 $f(x)=\left(x^2+3x-2\right)^4$ $\quad f'(x)=4\left(x^2+3x-2\right)^3(2x+3)$

Ejercicio 4:

1 $f(x)=\sqrt{x^2-2x+3}$ $\quad f'(x)=\frac{2x-2}{2\sqrt{x^2-2x+3}}=\frac{x-1}{\sqrt{x^2-2x+3}}$

2 $f(x)=\sqrt[4]{x^5-x^3-2}$ $\quad f'(x)=\frac{5x^4-3x^2}{4\sqrt[4]{\left(x^5-x^3-2\right)^3}}$

3 $f(x)=\sqrt[3]{\frac{x^2+1}{x^2-1}}$ $\quad f'(x)=\frac{\frac{2x\left(x^2-1\right)-\left(x^2+1\right)2x}{\left(x^2-1\right)^2}}{3\sqrt[3]{\left(\frac{x^2+1}{x^2-1}\right)^2}}=\frac{\frac{-4x}{\left(x^2-1\right)^2}}{3\sqrt[3]{\left(\frac{x^2+1}{x^2-1}\right)^2}}=\frac{-4x}{3\left(x^2-1\right)^2\sqrt[3]{\left(\frac{x^2+1}{x^2-1}\right)^2}}=$

$\frac{-4x}{3\sqrt[3]{\left(\frac{x^2+1}{x^2-1}\right)^2\left(x^2-1\right)^6}}=\frac{-4x}{3\sqrt[3]{\left(x^2+1\right)^2\left(x^2-1\right)^4}}=\frac{-4x}{3\sqrt[3]{\left(x^4-1\right)^2\left(x^2-1\right)^2}}$

Ejercicio 5:

1 $f(x)=10^{\sqrt{x}}$ $\quad f'(x)=\frac{1}{2\sqrt{x}}\cdot 10^{\sqrt{x}}\cdot \ln 10$

2 $f(x)=e^{3-x^2}$ $\quad f'(x)=-2x\cdot e^{3-x^2}$

3 $f(x)=\frac{e^x+e^{-x}}{2}$ $\quad f'(x)=\frac{e^x-e^{-x}}{2}$

4 $f(x)=3^{2x^2}\cdot\sqrt{x}$ $\quad f^{-1}(x)=4x\cdot 3^{2x^2}\cdot\ln 3\cdot\sqrt{x}+\frac{3^{2x^2}}{2\sqrt{x}}==3^{2x^2}\left(4x\cdot\sqrt{x}\cdot\ln 3+\frac{1}{2\sqrt{x}}\right)$

5 $f(x)=\frac{e^{2x}}{x^2}$ $\quad f'(x)=\frac{2\cdot e^{2x}\cdot x^2-e^{2x}\cdot 2x}{x^4}=\frac{2x\cdot e^{2x}(x-1)}{x^4}==\frac{2\cdot e^{2x}(x-1)}{x^3}$

Ejercicio 6:

1 $f(x)=\ln\left(2x^4-x^3+3x^2-3x\right)$ $\quad f'(x)=\frac{8x^3-3x^2+6x-3}{2x^4-x^3+3x^2-3x}$

2 Aplicando las **propiedades de los logaritmos** obtenemos: $f(x)=\ln\left(e^x+1\right)-\ln\left(e^x-1\right)$

$f'(x)=\frac{e^x}{e^x+1}-\frac{e^x}{e^x-1}=\frac{e^{2x}-e^x-e^{2x}-e^x}{\left(e^x+1\right)\left(e^x-1\right)}==\frac{2e^x}{c^{2x}-1}$

3 Aplicando las **propiedades de los logaritmos** obtenemos: $f(x)=\frac{1}{2}[\log(1+x)-\log(1-x)]$

$f'(x)=\frac{1}{2}\left(\frac{1}{1+x}-\frac{-1}{1-x}\right)\cdot\log e=\frac{1}{2}\cdot\frac{1-x+1+x}{1-x^2}\cdot\log e=$ $\quad =\frac{1}{1-x^2}\cdot\log e$

4 Aplicando las **propiedades de los logaritmos** obtenemos: $f(x)=\frac{1}{2}[\ln x+\ln(1-x)]$

$f'(x)=\frac{1}{2}\left(\frac{1}{x}+\frac{-1}{1-x}\right)=\frac{1}{2}\cdot\frac{1-x-x}{x(1-x)}=$ $\quad -\frac{1-2x}{2x(1-x)}$

5 Aplicando las **propiedades de los logaritmos** obtenemos: $f(x)=\frac{1}{3}[\ln 3x-\ln(x+2)]$

$f'(x)=\frac{1}{3}\left(\frac{3}{3x}-\frac{1}{x+2}\right)=\frac{1}{3}\cdot\frac{x+2-x}{x(x+2)}==\frac{2}{3x(x+2)}$